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Alternative Approaches to Testing Non-Nested Models with Autocorrelated Disturbances: An Application to Models of U.S. Unemployment

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Abstract

Since departures from the classical assumptions regarding the disturbances in a linear regression model arise frequently in empirical applications, several computationally straightforward procedures are presented in this paper for testing non–nested models when the disturbances of these models follow first– or higher–order autoregressive processes. An empirical example is used to illustrate how the procedures may be used to test competing Keynesian and New Classical non–nested models of unemployment for the U.S. using annual time series data for 1955–85.

1. Introduction

Specification tests have an important role to play in analyzing the adequacy of econometric models. One important area of research in econometric model evaluation is testing non–nested regression models. However, research on this topic has concentrated on models that satisfy the classical assumptions of serial independence, homoscedasticity and normality of the disturbances. Since departures from these classical assumptions arise frequently in empirical applications, it is essential to develop computationally straightforward procedures for testing general forms of non–nested econometric models. By a general form of a model is meant non–standard cases of non–nested linear regression models when some or all of the classical assumptions are violated, in particular, when there is serial correlation in the disturbances.

The paper is in two parts. The first part deals with problems associated with non–nested models when the disturbances follow first– or higher–order autoregressive processes. Since residual serial correlation may arise in a dynamic specification, lagged dependent variables are permitted. Several asymptotically valid procedures are presented and these may be computed from standard regression packages. An empirical example is presented in the second part of the paper to illustrate the use of the non–nested tests when at least one of the models under consideration exhibits autoregressive disturbances.

The plan of the paper is as follows. In Section 2 we discuss alternative asymptotic procedures that have been developed in the literature for testing non–nested regression models, and examine how they may be modified to take account of alternative autoregressive error specifications. The relations between some of these tests are emphasized to simplify the steps required in their computation. A straightforward generalization to take account of higher–order autoregressive error specifications is then presented. The methods developed in the paper are then used in Section 3 to test Keynesian and New Classical models of unemployment for the U.S. using annual data for the period 1955–85. Some concluding comments are given in Section 4.

2. Test Procedures for Models with Serial Correlation

In the case of non–nested linear regression models with normal and spherical errors, the null model H_0 is tested against the alternative model H_1 and the two models are specified as

$$H_0 : y = X\beta + u_0 , \quad u_0 \sim N(0, \sigma_0^2 I_n)$$

$$H_1 : y = Z\gamma + u_1 , \quad u_1 \sim N(0, \sigma_1^2 I_n)$$

in which y is the $n \times 1$ vector of observations on the dependent variable, X and Z are $n \times k$ and $n \times g$ matrices of observations on k and g linearly independent regressors, β and γ are $k \times 1$ and $g \times 1$ vectors of unknown parameters, and u_0 and u_1 are $n \times 1$ vectors of normally, independently and identically distributed disturbances. Strictly speaking, normality is not required for all the tests to be discussed below, but it will prove convenient in what follows. It is also assumed that X and Z are not orthogonal, and that the limits of $n^{-1}X'X$, $n^{-1}Z'Z$ and $n^{-1}X'Z$ exist, with the first two positive definite and the third non–zero. If X and Z contain stochastic rather than fixed elements, the probability limits of the appropriate matrices must exist, and X and Z must be distributed independently of u_0 and u_1.

In order to take account of departures from sphericality, the two models may be rewritten conveniently as

$$H_0 : y_t = x_t'\beta + u_{0t} \tag{1}$$

$$H_1 : y_t = z_t'\gamma + u_{1t} \tag{2}$$

in which x_t' and z_t' are the t'th rows of X and Z, respectively, and $t = 1, 2, ..., n$. In practice, departures from the classical assumptions can arise in one of two ways. Economic theory may postulate that the errors in a model follow an autoregressive process of specified order. Alternatively, the order of the process may be determined empirically through the use of diagnostic checks and/or information criteria. Regardless of the ways in which the presence and the order of the autoregressive process are detected, autoregressive errors are frequently presented as an integral part of a linear regression model. Extending this line of argument to the non–nested case, such models may themselves display different autoregressive patterns.

Pesaran (1974) derived a test of a linear regression model against a non–nested linear alternative when the regressors of both models are non–stochastic and the errors follow stationary first–order autoregressive (AR(1)) processes, namely

$$u_{it} = \rho_i u_{it-1} + \epsilon_{it}, \quad \epsilon_{it} \sim \mathrm{NID}(0, \sigma_i^2), \quad |\rho_i| < 1 \tag{3}$$

for $i = 0, 1$ and $t = 2, 3, \ldots n$. The approach was based on a direct application of Cox's (1961, 1962) procedure for testing separate families of hypotheses. While Pesaran's test is not as straightforward computationally as some of the procedures to be discussed below, and modifications are required to account for lagged dependent variables, it does have an advantage in finite samples over the other procedures in that it allows for *stochastic* (rather than *fixed*) initial values of the AR(1) processes involved. Of course, initial conditions have no effect on the properties of the tests asymptotically and would be of concern primarily in finite samples.

Concentrating solely on perceived computational advantages, and extending the framework of straightforward test procedures without due care and attention, can have some undesirable consequences. A simple illustration will suffice. There have been several direct applications of Davidson and MacKinnon's (1981) J test procedure to models exhibiting serially correlated errors (see e.g. Thornton (1985) and Johannes and Nasseh (1985)), although the original test was derived and intended for models with serially uncorrelated errors. A J–type test of H_0 against H_1, where each set of errors obeys (3), has sometimes been interpreted as being the test of $\lambda = 0$ in the auxiliary regression

$$y_t = x_t'b + \lambda\hat{y}_{1t} + v_t \tag{4}$$

in which

$$\hat{y}_{1t} = z_t'\hat{\gamma} + \hat{\rho}_1\hat{u}_{1t-1} = z_t'\hat{\gamma} + \hat{\rho}_1(y_{t-1} - z_{t-1}'\hat{\gamma})$$

is a consistent estimate of the predicted value of y_t under H_1 and $b = (1-\lambda)\beta$. Since v_t in (4) corresponds to u_{0t} under $H_0 : \lambda = 0$, it is tempting to infer that a direct extension of the J test procedure to models with serial correlation is valid, namely, presuming that v_t follows an AR(1) process for purposes of testing H_0 against H_1 via a test of $\lambda = 0$ in (4).

What is deceptive about such an approach? Observe that H_0 and H_1 may be rewritten as

$$H_0 : y_t = \rho_0 y_{t-1} + (x_t - \rho_0 x_{t-1})'\beta + \epsilon_{0t} = f_t(\beta, \rho_0) + \epsilon_{0t} \tag{5}$$

$$H_1 : y_t = \rho_1 y_{t-1} + (z_t - \rho_1 z_{t-1})'\gamma + \epsilon_{1t} = g_t(\gamma, \rho_1) + \epsilon_{1t}\,. \tag{6}$$

A linear combination of (1) and (6), with weights $(1-\lambda)$ and λ, respectively, together with the addition and subtraction of $\lambda\hat{y}_{1t}$, leads to the auxiliary regression

$$y_t = (1-\lambda)x_t'\beta + \lambda\hat{y}_{1t} + v_t \tag{7}$$

in which

$$v_t = (1-\lambda)u_{0t} + \lambda[\epsilon_{1t} + (g_t - \hat{y}_{1t})]$$

and $g_t \equiv g_t(\gamma, \rho_1)$. It can easily be seen that, even under $H_0 : \lambda = 0$, the disturbance term v_t in (7) will be asymptotically correlated with $\hat{y}_{1t}$. Observing that $\hat{y}_{1t}$ can be rewritten under H_0 as

$$\hat{y}_{1t} = z_t'\hat{\gamma} + \hat{\rho}_1(x_{t-1}'\beta - z_{t-1}'\hat{\gamma} + u_{0t-1})$$

and denoting the probability limit of $\hat{\rho}_1$ under H_0 by ρ_{10} (i.e. $\operatorname{plim}_{n\to\infty 0}\, \hat{\rho}_1 = \rho_{10}$), it is straightforward to show that

$$\operatorname*{plim}_{n\to\infty}{}_0 [n^{-1} \sum_{t=2}^{n} \hat{y}_{1t} v_t] = \sigma_0^2 \rho_0 \rho_{10}/(1-\rho_0^2) .$$

When the errors under H_0 and H_1 are serially correlated, $\hat{y}_{1t}$ and v_t will also be correlated. Thus, if serial correlation is observed in competing non–nested regression models, the use of ordinary least squares to test $\lambda = 0$ in (7), as in Backus (1984) and Milbourne (1985), would lead to biased test statistics. Moreover, the fact that $\hat{y}_{1t}$ and v_t are correlated implies that an efficient method of estimating ρ_0 is required. As a consequence, the Cochrane–Orcutt procedure will not be valid unless it uses a consistent estimator of ρ_0 at the initial stage. An appropriate procedure for estimating λ and testing $\lambda = 0$ in (4) under the assumption that v_t follows an AR(1) process is Hatanaka's (1974) two–step estimator, which is valid even in the presence of lagged dependent variables in both models.

An alternative procedure for testing H_0 against H_1 which would be straightforward to apply in the present context is to test $H_0 : c = 0$ in the comprehensive model formed from H_0 and H_1, namely

$$y_t = x_t'b + z_t'c + u_t \tag{8}$$

in which $b = (1 - \lambda)\beta$, $c = \lambda\gamma$, and $u_t = (1 - \lambda)u_{0t} + \lambda u_{1t}$, which follows an AR(1) process under $H_0 : \lambda = 0$. It is now well known for the case where both u_{0t} and u_{1t} are serially independent that the standard F test of $H_0 : c = 0$ in (8) yields a valid test of H_0 against H_1. Deaton (1982), Dastoor (1983) and Gourieroux, Monfort and Trognon (1983) derived a non–nested F test based on pseudo–true values of selected parameters of interest, McAleer and Pesaran (1986) showed that a similar analysis could be conducted using Roy's union–intersection principle, and Mizon and Richard (1986) based their F test on the encompassing principle. However, the F test has reduced asymptotic power compared with Cox–type tests (see Pesaran (1982a)) and also has smaller empirical power in finite samples (for further details, see Godfrey and Pesaran (1983)). The generalization of the F test of $c = 0$ in the case of serially correlated disturbances is a much more complicated matter. It is clear that, for an arbitrary value of λ, the composite disturbance u_t will not follow an autoregressive process, and maximum likelihood estimation of the parameters in (8) will not be a straightforward matter.

On the basis of the discussion presented above, H_0 may be tested against H_1 after transforming the models to their non–linear counterparts given in (5) and (6), respectively. For example, it is valid to apply the comprehensive model approach, or the procedures advanced by Davidson and MacKinnon (1981) and Fisher and McAleer (1981), to the non–linear models given in (5) and (6) because the disturbances are serially independent. Thus, (7) may be rewritten as

$$y_t = (1-\lambda)(\rho_0 y_{t-1} + x_t'\beta - \rho_0 x_{t-1}'\beta) + \lambda g_t + \epsilon_t \tag{9}$$

where $\epsilon_t = (1-\lambda)\epsilon_{0t} + \lambda\epsilon_{1t}$. The unobserved variable g_t may be replaced by any consistent estimator of its components under H_0. An obvious candidate is $\hat{g}_t = \hat{y}_{1t} = z_t'\hat{\gamma} + \hat{\rho}_1\hat{u}_{1t-1}$, an estimate of the conditional expectation of y_t under H_1, and this leads to the J test of $\lambda = 0$. If $\hat{y}_{10t}$ were to be substituted for g_t, where $\hat{y}_{10t}$ denotes the predictions from (6) when y_t is replaced by $\hat{y}_{0t} = x_t'\hat{\beta} + \hat{\rho}_0\hat{u}_{0t-1}$, the predictions from H_0, this would give the JA test of Fisher and McAleer (1981). In each case it should be noted that, unlike (7), the disturbance term in (9) will be asymptotically uncorrelated with g_t, or a consistent estimator substituted in its place. However, estimation of λ and testing $\lambda = 0$ in (9) involves non–linear restrictions between the parameters $(1-\lambda)\rho_0$, $(1-\lambda)\beta$, $-\rho_0(1-\lambda)\beta$ and λ. This means that a test of $\lambda = 0$ in (9) generally involves non–linear estimation and the testing of H_0 by this method on standard computer packages may not always be possible.

A computationally less demanding procedure which does not require non–linear estimation would be to apply the P test of Davidson and MacKinnon (1981) directly to (5) and (6) viewed as two non–nested non–linear regression models. The appropriate test statistic is computed as the usual t–ratio of the estimate of λ in the auxiliary linear regression

$$\hat{\epsilon}_{0t} = \hat{F}_t' d + \lambda(\hat{\epsilon}_{0t} - \hat{\epsilon}_{1t}) + \epsilon_t \tag{10}$$

in which $\hat{\epsilon}_{it} = y_t - \hat{y}_{it}$ denote the prediction errors under H_i $(i = 0, 1)$, $\hat{F}_t$ denotes the maximum likelihood estimates, under H_0, of the partial derivatives of $f_t(\beta, \rho_0)$ in (5) with respect to β and ρ_0, namely

$$\hat{F}_t' = (\partial\hat{f}_t/\partial\beta', \partial\hat{f}_t/\partial\rho_0) = [(x_t - \hat{\rho}_0 x_{t-1})', \hat{u}_{0t-1}] \tag{11}$$

$$\hat{u}_{0t-1} = y_{t-1} - x_{t-1}'\hat{\beta}$$

and

$$d' = [(\beta - \hat{\beta})', \rho_0 - \hat{\rho}_0].$$

Noting that

$$y_t = \hat{y}_{0t} + \hat{\epsilon}_{0t} = \hat{y}_{1t} + \hat{\epsilon}_{1t}$$

it follows that $\hat{\epsilon}_{0t} - \hat{\epsilon}_{1t} = \hat{y}_{1t} - \hat{y}_{0t}$, so that the difference in the prediction errors is equal to the negative of the difference in the predictions of the two models. Defining

$$\hat{\eta}_{10t} \equiv \hat{\epsilon}_{1t} - \hat{\epsilon}_{0t},$$

substitution of (11) into (10) yields, after some algebraic manipulation, the following convenient auxiliary linear regression, namely

$$y_{*t} = x_{*t}'\beta + \delta\hat{u}_{0t-1} - \lambda\hat{\eta}_{10t} + \epsilon_t \tag{12}$$

in which $y_{*t} = y_t - \hat{\rho}_0 y_{t-1}$, $x_{*t} = x_t - \hat{\rho}_0 x_{t-1}$ and $\delta = \rho_0 - \hat{\rho}_0$. The expression in (12) is didactically more appealing than that in (10) because, under $H_0 : \lambda = 0$, the regression of y_{*t} on the elements of x_{*t}' and on $\hat{u}_{0t-1}$ is Hatanaka's (1974) two–step (or residual–adjusted Aitken) estimator for the dynamic adjustment model with autoregressive errors. In the context of this problem, then, a test of significance on $\hat{\eta}_{10t}$ in (12) is equivalent to a test of H_0 given in (5) against H_1 in (6).

A similar result can also be obtained through the application of the Cox test of Pesaran and Deaton (1978) directly to (5) and (6). The Cox test statistic is given by

$$N_0 = T_0/[\hat{V}_0(T_0)]^{\frac{1}{2}} \tag{13}$$

in which

$$T_0 = (n/2) \log (\hat{\sigma}_1^2/\hat{\sigma}_{10}^2) \tag{14}$$

$$\hat{V}_0(T_0) = (\hat{\sigma}_0^2/\hat{\sigma}_{10}^4)[(\hat{y}_0 - \hat{y}_{10})' M_{\hat{F}}(\hat{y}_0 - \hat{y}_{10})] \tag{15}$$

$$\hat{\sigma}_{10}^2 = \hat{\sigma}_0^2 + n^{-1}(\hat{y}_0 - \hat{y}_{10})'(\hat{y}_0 - \hat{y}_{10}) \tag{16}$$

$$M_{\hat{F}} = I - \hat{F}(\hat{F}'\hat{F})^{-1}\hat{F}'$$

the t'th row of $\hat{F}$ is given by $\hat{F}_t'$ in (11), and $\hat{y}_{10}$ denotes the n×1 vector of estimates of the expectations of y_t under H_1 when y_t is replaced by $\hat{y}_{0t}$ in (6), where $\hat{y}_{0t}$ is given by

$$\hat{y}_{0t} = x_t'\hat{\beta} + \hat{\rho}_0\hat{u}_{0t-1} = x_t'\hat{\beta} + \hat{\rho}_0(y_{t-1} - x_{t-1}'\hat{\beta}).$$

Given the results of Lemma 3 of Davidson and MacKinnon (1981), it is easily established that the N_0 test in (13) and the tests of $\lambda = 0$ in (10) and (12) are asymptotically equivalent under H_0.

Since it is asymptotically valid under H_0 to replace $\hat{y}_{10}$ in (15) and (16) with $\hat{y}_1$, an asymptotically equivalent expression for (14) under H_0 is given by

$$T_0 = (n/2) \log [\hat{\epsilon}_1'\hat{\epsilon}_1/(\hat{\epsilon}_0'\hat{\epsilon}_0 + \hat{\eta}_{10}'\hat{\eta}_{10})]$$

or

$$T_0 = -(n/2) \log (1 - 2\hat{\epsilon}_0'\hat{\eta}_{10}/\hat{\epsilon}_1'\hat{\epsilon}_1) \tag{17}$$

in which $\hat{\eta}_{10} = \hat{\epsilon}_1 - \hat{\epsilon}_0$. A first–order linearization of T_0 in (17) is given by

$$T_0 = \hat{\epsilon}_0' \hat{\eta}_{10} / \hat{\sigma}_1^2 . \tag{18}$$

As the expressions in (14) and (18) are asymptotically equivalent under H_0, either may be substituted into (13) to yield the Cox test statistic. However, it is possible to derive the asymptotic distribution of $n^{-\frac{1}{2}} \hat{\epsilon}_0' \hat{\eta}_{10}$ under H_0, and to base a Cox–type test on $\hat{\epsilon}_0' \hat{\eta}_{10}$ directly. Suppose (5) and (6) are rewritten as

$$H_0 : y_t = f_t(\beta, \rho_0) + \epsilon_{0t} = f_t(\theta) + \epsilon_{0t}$$

$$H_1 : y_t = g_t(\gamma, \rho_1) + \epsilon_{1t} = g_t(\phi) + \epsilon_{1t}$$

with maximum likelihood estimates of $\theta' = (\beta', \rho_0)$ and $\phi' = (\gamma', \rho_1)$ denoted as $\hat{\theta}$ and $\hat{\phi}$ under H_0 and H_1, respectively. If $(\hat{\theta}', \hat{\phi}')'$ converges in probability under H_0 to $(\theta_0', \phi_*')'$, it is possible to rewrite $\hat{\epsilon}_0' \hat{\eta}_{10}$ as

$$\hat{\epsilon}_0' \hat{\eta}_{10} = \epsilon_0' M_F \eta_{10} + o_p(1)$$

in which $\eta_{10} = f(\theta_0) - g(\phi_*)$. Under suitable regularity conditions (see e.g. White (1982)), it is straightforward to show that, when H_0 holds,

$$n^{-\frac{1}{2}} \hat{\epsilon}_0' \hat{\eta}_{10} \overset{a}{\sim} N(0, \sigma_0^2 \operatorname*{plim}_{n\to\infty} n^{-1} \eta_{10}' M_F \eta_{10}).$$

Therefore, both the Cox and P tests are asymptotically equivalent under H_0 to the following linearized version of the Cox test, namely

$$LN_0 = \hat{\epsilon}_0' \hat{\eta}_{10} / [\hat{\sigma}_0^2 (\hat{\eta}_{10}' M_{\hat{F}} \hat{\eta}_{10})]^{\frac{1}{2}} \tag{19}$$

in which $\hat{\sigma}_0^2 = n^{-1} \hat{\epsilon}_0' \hat{\epsilon}_0$ is the maximum likelihood estimate of σ_0^2 under H_0 and $\hat{\eta}_{10} = \hat{\epsilon}_1 - \hat{\epsilon}_0$ is a consistent estimate of η_{10} under H_0. Notice that the denominator of LN_0 in (19) can be computed conveniently as the residual sum of squares from the least squares regression of $\hat{\eta}_{10}$ on the columns of $\hat{F}$.

The two tests in equations (12) and (19) would be expected to have different properties in small samples, even though they are asymptotically equivalent. Recently, Bernanke et al. (1988) have used several tests, including tests which were developed in an earlier version of the present paper, to test non–nested models of investment subject to serial correlation. On the basis of Monte Carlo experiments, Bernanke et al. (1988, p. 320) suggest that the P test given in equation (12) appears to be the best of those presently available.

The results obtained above can be generalized straightforwardly to the case where the disturbance u_{it} $(i = 0, 1)$ follows a stationary autoregressive process of order p_i, namely $AR(p_i)$:

$$u_{it} = \sum_{j=1}^{p_i} \rho_{ij} u_{it-j} + \epsilon_{it}, \quad i = 0, 1 \tag{20}$$

where $t = p+1, p+2, ..., n$ and $p = \max(p_0, p_1)$. In this general setting, the P test of H_0 is simply a test of $\lambda = 0$ in the auxiliary linear regression

$$y_{*t} = x'_{*t}\beta + \sum_{j=1}^{p_0} \delta_j \hat{u}_{0t-j} - \lambda\hat{\eta}_{10t} + \epsilon_t$$

in which

$$y_{*t} = y_t - \sum_{j=1}^{p_0} \hat{\rho}_{0j} y_{t-j}$$

$$x_{*t} = x_t - \sum_{j=1}^{p_0} \hat{\rho}_{0j} x_{t-j}$$

$$\delta_j = \rho_{0j} - \hat{\rho}_{0j}$$

$$\hat{\eta}_{10t} = \hat{y}_{1t} - \hat{y}_{0t}$$

$$\hat{y}_{0t} = x_t'\hat{\beta} + \sum_{j=1}^{p_0} \hat{\rho}_{0j}\hat{u}_{0t-j}$$

and

$$\hat{y}_{1t} = z_t'\hat{\gamma} + \sum_{j=1}^{p_1} \hat{\rho}_{1j}\hat{u}_{1t-j}\,.$$

The Cox test of Pesaran and Deaton (1978) will have the same form as in (13), with $\hat{F}_t'$ in (11) replaced by

$$\hat{F}_t' = (x_{*t}', \hat{u}_{0t-1}, \hat{u}_{0t-2}, \ldots, \hat{u}_{0t-p_0})\,.$$

It is also straightforward to obtain a linearized Cox test as in (19) corresponding to the case where u_{it} follows an AR(p_i) process.

So far in this section we have considered cases in which the disturbances have been serially correlated. The situation would be made considerably simpler if the errors were to be independent but not identically distributed. Unless the actual form of the heteroscedasticity were known, transformation of (1) and (2) along the lines given in (5) and (6) (i.e. in the case of *known* autoregressive structures) would not be possible. Nevertheless, the J, JA and F tests of H_0 may be constructed in the form of (4) or (8), where the form of heteroscedasticity of u_t is *unknown*, if White's (1980) 'heteroscedasticity–consistent variance estimator' is used. Thus, it is entirely straightforward to test two non–nested linear regression models against each other in the presence of general forms of heteroscedasticity, since there exist computationally straightforward non–nested tests which require estimation only of an auxiliary linear regression.

3. Models of U.S. Unemployment

In this section an empirical example is used to illustrate the application of the non–nested P and LN test procedures, as given in equations (12) and (19), respectively.

The example centres on the debate between the New Classical model of unemployment for the U.S. as developed in Barro (1977, 1979, 1981) and Rush and Waldo (1988), and the Keynesian (or activist) model of Pesaran (1982b, 1988). The Keynesian and New Classical models are given as follows:

Keynesian model: Pesaran (1988, Appendix Table 2)

$$UN_t = \psi_0 + \psi_1 MIL_t + \psi_2 UN_{t-1} + \psi_3 DM_t + \psi_4 DM_{t-1} + \psi_5 DM_{t-2} + \psi_6 t + \psi_7 WAR_t + \epsilon_{tK} \quad (21)$$

New Classical model: Barro (1977), Pesaran (1982b, 1988), Rush and Waldo (1988)

$$UN_t = \alpha_0 + \alpha_1 MIL_t + \alpha_2 MINW_t + \alpha_3 DMRH_t + \alpha_4 DMRH_{t-1} + \alpha_5 DMRH_{t-2} + \epsilon_{tNC} \quad (22)$$

in which $DMRH_t = DM_t - E_{t-1}(DM_t)$ is the error term in the money supply equation given by

$$DM_t = \beta_0 + \beta_1 DM_{t-1} + \beta_2 DM_{t-2} + \beta_3 UN_{t-1} + \beta_4 E_{t-1}(FEDV_t) + DMRH_t \quad (23)$$

where $E_{t-1}(FEDV_t) = FEDV_t - 0.8DGR_t$ and $DGR_t = DG_t - E_{t-1}(DG_t)$ is the error term in the government expenditure equation given by[1]

$$DG_t = \gamma_0 + \gamma_1 DG_{t-1} + \gamma_2 UN_{t-1} + \gamma_3 WAR_t + DGR_t. \quad (24)$$

The variables in equations (21) – (24) have the following definitions:

$UN_t = \log[U_t/(1 - U_t)]$

U_t = annual average unemployment rate

MIL_t = measure of military conscription

[1] Note that, in defining DGR_t by (24), it is implicity assumed that the value of WAR_t is known to economic agents at time t–1, that is, WAR_t is perfectly predictable at time t–1.

$MINW_t$ = minimum wage variable

DM_t = rate of growth of money supply (M1 definition)

$DMRH_t = DM_t - E_{t-1}(DM_t)$ = unanticipated rate of growth of money supply

$FEDV_t$ = real federal government expenditure relative to its normal level

$E_{t-1}(FEDV_t)$ = anticipated value of $FEDV_t$ formed at time t–1

DG_t = rate of growth of real federal government expenditure

$DGR_t = DG_t - E_{t-1}(DG_t)$ = unanticipated rate of growth of real federal government expenditure

WAR_t = a step dummy variable measuring the intensities of different wars

t = time trend.

The primary purpose of these two empirical models is to explain the rate of unemployment in the U.S.. Fiscal and monetary variables are included in the reduced form Keynesian model, together with a time trend to explain gradual changes in the natural rate of unemployment over time. All variables in the Keynesian model are observable and may be efficiently estimated by ordinary least squares (OLS) if the model is correctly specified. However, since the New Classical model states that only unanticipated changes in the money supply affect the unemployment rate, a sequential estimation and testing procedure may be used to generate the unobserved money supply shocks as the OLS residuals from the money supply equation in (23). Barro (1977, 1979, 1981) assumes that real federal government expenditure relative to its normal level, $FEDV_t$, can be anticipated perfectly at time t–1 (that is, $E_{t-1}(FEDV_t) = FEDV_t$). When this unrealistic assumption is relaxed, the expected value of $FEDV_t$ may be estimated by using the OLS residuals from the government expenditure equation in (24). The variables in the New Classical model include monetary shocks, real variables to explain the natural rate of unemployment, countercyclical monetary and fiscal policy variables, and variables to account for government financing needs.

Maximum likelihood methods may be used to estimate the New Classical model (that is, equations (22)–(24)), as in McAleer and McKenzie (1989), or alternatively a sequential procedure may be used, as in virtually all of the papers in the literature. Using OLS to estimate equations (24), (23) and (22) in a sequential manner generally yields inefficient estimators and incorrect standard errors. Pagan (1984, 1986) and Pesaran (1987, chapter 7) provide a detailed analysis of these two issues. Murphy and Topel (1985) also examine the problem of incorrect standard errors, and McAleer and McKenzie (1988) present very simple proofs of several of the existing efficiency results. The specific econometric problems associated with the three–equation New Classical model are analysed in McAleer and McKenzie (1989, Appendix A). For present purposes, it is sufficient to state that sequential OLS estimation yields inefficient estimators and the standard errors are understated, so that t–ratios are biased towards rejection of the relevant null hypotheses.

Selection of the sample period is problematical. Barro (1977) used annual data for the U.S. for 1946–73, Barro (1979) used data for 1946–77 and 1949–77 (the latter to avoid problems in the immediate post–war years for the unemployment equation), and Barro (1981) chose 1946–78 (although the sample period became 1947–78 when a correction was made for first–order autoregressive errors in the unemployment equation) and 1949–78. Pesaran (1982b) used data for 1946–73; Rush and Waldo (1988) chose 1946–73, 1949–73, 1946–85 and 1949–85, and noted some evidence of serial correlation for the extended sample period; and Pesaran (1988) used 1946–73 and 1946–85. The sample period used in this paper is 1955–85. This covers a more recent period than that considered by Barro in his three papers, and also avoids the major dispute between Rush and Waldo (1988) and Pesaran (1982b, 1988) regarding the use of a dummy variable for war by the former authors to accommodate the situation whereby the public will anticipate and has knowledge of the quantitative effect of an abrupt reduction in government military spending when a war ends.

Both estimation and testing were undertaken using the computer package Microfit (see Pesaran and Pesaran (1989)). The OLS estimates of the Keynesian model are given in Table 1. The signs and magnitudes of the estimated coefficients are in general agreement with those of Pesaran (1988) for 1946–85. Since the sample period omits the effects of World War II and the Korean War, it is not surprising that the war dummy variable is not statistically significant. Moreover, the second lag of the money supply growth rate is not significant. Deleting the two insignificant variables leaves all other estimates virtually unchanged and has no discernible effect on the outcome of the non–nested test statistics reported in Table 3. Finally, there appear to be no significant departures from the classical assumptions of correct model specification or from normally, independently and identically distributed errors.

Estimates of the New Classical unemployment equation with AR(1) errors are presented in Table 2. The government expenditure and money supply growth equations are estimated sequentially by OLS, while the unemployment equation is estimated by exact maximum likelihood subject to a first–order autoregressive error process. Apart from the correction for serial correlation, the estimated magnitudes and signs of the parameters are very similar to results available in the literature, including the insignificance of the minimum wage variable. The diagnostic tests for the New Classical model are calculated using the adjusted residuals obtained from the Cochrane–Orcutt transformation and the first derivatives of the non–linear function resulting from the AR(1) errors. The classical assumptions regarding the errors and correct model specification appear to be satisfied for the New Classical unemployment equation.

The results from testing the non–nested Keynesian and New Classical models against each other using the P and LN tests are reported in Table 3. As shown in Pesaran (1988) and McAleer and McKenzie (1989), the non–rejection of the Keynesian model is supported strongly. On the other hand, the calculated test statistics for the New Classical model are beyond conventional critical values, leading to rejection, although it should be noted that the standard errors for the New Classical model are understated so that the rejection may be problematical.

4. Conclusion

In this paper we have presented some asymptotically valid and computationally straightforward procedures for testing non–nested regression models with first–order autoregressive disturbances. The procedures were also generalized to take account of higher–order autoregressive processes in a simple manner. An empirical example concerning Keynesian and New Classical explanations of unemployment for the U.S. using annual data for 1955–85 was presented to illustrate the use of the testing procedures.

TABLE 1

OLS Estimation of the Keynesian Model, 1955–85

Regressor	Coefficient	Standard error	t–ratio
INTERCEPT	–2.0292	.5018	–4.0439
MIL	–4.0154	1.1387	–3.5264
UN(–1)	.3743	.1255	2.9815
DM	–5.4002	1.7089	–3.1600
DM(–1)	–8.5503	1.8801	–4.5477
DM(–2)	–1.7311	2.2839	– .7580
TREND	.0365	.0112	3.2624
WAR	–.2123	.2146	–.9894

R–squared	.8945	F–statistic F(7,23)	27.8537
R–bar–squared	.8624	S.E. of regression	.1134
Residual sum of squares	.2958	Mean of dependent variable*	–2.8340
S.D. of dependent variable	.3057	Maximum of log–likelihood	28.1187
DW–statistic	1.9735	Durbins′ h–statistic	.1033

Notes: 1 Thirty–one observations used for estimation from 1955 to 1985.

* Dependent variable is UN.

Diagnostic tests

Test statistics	LM version	F version
(A) Serial correlation	Chi–sq.(1) = .0043	F(1, 22) = .0030
(B) Functional form	Chi–sq.(1) = .1063	F(1, 22) = .0757
(C) Normality	Chi–sq.(2) = 2.2514	Not applicable
(D) Heteroscedasticity	Chi–sq.(1) = 1.9855	F(1, 29) =1.9845

(A) Lagrange multiplier test of residual serial correlation.

(B) Ramsey′s RESET test using the square of the fitted values.

(C) Based on a test of skewness and kurtosis of residuals.

(D) Based on the regression of squared residuals on squared fitted values.

TABLE 2

Estimation of the New Classical Model with AR(1) Errors, 1955–85

Regressor	Coefficient	Standard error	t–ratio
INTERCEPT	–2.7629	.2777	–9.9501
MIL	–6.2247	1.1653	–5.3418
MINW	.3991	.6849	.5826
DMRH	–3.5209	1.8827	–1.8701
DMRH(–1)	–9.5880	1.9896	–4.8190
DMRH(–2)	–6.0384	2.1697	–2.7830

R–squared	.8509	F–statistic F(6,24)	22.825[illegible]
R–bar–squared	.8136	S.E. of regression	.1320
Residual sum of squares	.4180	Mean of dependent variable*	–2.8340
S.D. of dependent variable	.3057	Maximum of log–likelihood	22.6659
DW–statistic	1.9021		

Parameters of the autoregressive error specification

$$\hat{u} = \underset{(2.5073)}{.4106}\ \hat{u}(-1) + \hat{\epsilon}$$

t–ratio based on asymptotic standard error is given in brackets

Log–likelihood ratio test of AR(1) relative to OLS: Chi–sq. (1) = 5.1446

Notes:
1 The exact inverse interpolation method converged after 5 iterations.
2 Thirty–one observations used for estimation from 1955 to 1985.
* Dependent variable is UN.

TABLE 2 (continued)

Diagnostic tests

Test statistics	LM version	F version
(A) Serial correlation	Chi–sq.(1) = .0002	F(1, 22) = .0002
(B) Functional form	Chi–sq.(1) = .5508	F(1, 22) = .4115
(C) Normality	Chi–sq.(2) = 2.6699	Not applicable
(D) Heteroscedasticity	Chi–sq.(1) = 3.2371	F(1, 28) =3.3868

(A) Lagrange multiplier test of residual serial correlation.

(B) Ramsey's RESET test using the square of the fitted values.

(C) Based on a test of skewness and kurtosis of residuals.

(D) Based on the regression of squared residuals on squared fitted values.

TABLE 3

Non–nested Test Statistics

Null Model	Alternative Model	P test Equation (12)	LN test Equation (19)
Keynesian	New Classical	–.6948	.7031
New Classical	Keynesian	3.5583	–3.3197

Note: The P and LN test statistics are asymptotically distributed under the null hypothesis as N(0, 1).

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